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Normal, Gamma, and Beta Distributions

The normal probability distribution is the most widely used one out of the three and is typically represented in a bell-shaped curve when graphed. When a random variable Y is said to have a normal probability distribution, and σ > 0 and −∞ < µ < ∞, the density function for Y is the following:

, −∞ < y < ∞

The normal density function contains two parameters, both µ and σ. In this case, the expected value, or E(Y), is equal to µ and the variance, or V(Y), is equal to σ2. When graphed, µ is located at the center of the distribution and represents the mean of the data, and σ depicts the spread of the data.

When determining the area under the bell curve corresponding to P(a ≤ Y ≤ b), it will require finding the integral of f(y) as follows:

The closed form expression for this integral does not exist, so the use of R and S-plus will likely be required in order to find the probabilities for these random variables. These techniques can typically be done using the *pnorm* command that can be found on numerous graphing calculators.

With regards to the normal density function, when graphed as a normal distribution, it is symmetrical around the value µ and the tabulated areas of the graph (shown on figure 4.11 on page 180 in the textbook) is typically only on one side of the curve and to the right of point z, which is the distance from the mean that is measured in standard deviations. To calculate z, the following formula will need to be calculated:

Typically, you will need to find the innermost z, or z1, and subtract it from the outermost border of z, or z2, in order to find the area within z.

Also, a normal random variable Y can also be transformed to a standard normal random variable Z by using this formula:

Z represents a point measured from the mean of a normal random variable, with the distance expressed in standard deviation of the original normal random variable. Therefore, the mean value of Z has to be 0, and the standard deviation is also 1.

Relative to normal distributions, the gamma probability distribution takes into account of some random variables that are always nonnegative, and therefore, yield distributions of data that skew towards to the right. This typically results in a graphed distribution taking on a more nonsymmetrical look. This also indicates that the majority of the area under the density function will be located near the origin while also dropping gradually as y increases. A random variable Y that has a gamma distribution with parameters α > 0 and β > 0 can be represented as such:

The quantity is known as the gamma function in this case, where α is considered the shape parameter because of its relationship to what the shape of a gamma distribution will ultimately look like depending on what the value of α is. For parameter β, it is called the scale parameter because multiplying a gamma-distributed random variable by a positive constant, changes the scale on which a measurement can be made and will produce a random variable that has a constant value of α while altering the value of β.

To compute the mean and variance of gamma-distributed random variables, you would need to apply the following:

and

In one special case of gamma-distributed random variables, the random variable is said to have a chi-square distribution with v degrees of freedom, represented as a positive integer, and parameters α = ν/2 and β = 2. And in this case, to compute the mean and variance is as follows:

and

In another special case, a gamma density function where α = 1 is considered an exponential density function. With parameter β > 0, it can be computed with the following:

In this case, to find the mean and the variance, compute the following:

and

The last distribution, the beta probability distribution, is a 2-parameter density function defined over a closed interval of 0 ≤ y ≤ 1. With parameters α > 0 and β > 0, the density function of random variable Y is:

where..

When graphed, beta density functions will take on many differing shapes when given various values for the parameters α and β.

To find the mean and variance of this distribution, with parameters α > 0 and β > 0, it is as follows: